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Incoherent scattering of 279 keV gamma rays by K-shell electrons

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Abstract. The differential scattering cross section of 279 keV gamma rays incoherently scattered by the K-shell electrons of tungsten and erbium are experimentally determined and expressed in terms of the free electron scattering cross section. Two scintillation spectrometers and a fast–slow coincidence circuit are employed. The scattered gamma ray spectrum is displayed on a twenty channel analyser. The experimental results are compared with those computed on the basis of different theories and the other available experimental values.

1. Introduction

The incoherent scattering of 662 keV gamma rays by the K-shell electrons of high Z elements has been studied by some workers (Brini *et al* 1960, Motz and Missoni 1961, Sujkowski and Nagel 1961, Varma and Eswaran 1962, Shimizu *et al* 1965, Ramalinga Reddy *et al* 1966, Pingot 1968, East and Lewis 1969, Krishna Reddy *et al* 1970) while studies using low energy gamma rays are very meagre. Ramalinga Reddy *et al* (1967) used 320 keV gamma rays from a ^{51}Cr source to study the differential cross section ratio with lead, tantalum and samarium scatterers at scattering angles of 45°, 60°, 90° and 110° and Pingot (1969) used 279 keV gamma rays and a gold target for the same studies at angles of 20°, 55°, 90°, 125° and 160°. The present work was undertaken by the authors to determine the differential cross section of 279 keV gamma rays incoherently scattered by the K-shell electrons of tungsten and erbium.

2. Experimental set-up

The incoherent scattering of a gamma ray by a K-shell electron leads to the ejection of the K electron from its orbit, leaving a vacancy in the K-shell. The vacancy is filled in a very short time by transitions from the outer shells and isotropic emission of a K x ray takes place with a certain probability defined by the K-shell fluorescent yield. This situation is utilized to select the events in which the gamma rays scattered inelastically by K-shell electrons are detected in coincidence with the fluorescent K x ray thereby eliminating the unwanted events.

The experimental set-up resembles the one previously described (Krishna Reddy *et al* 1970) except for a change in the orientation of the target. In the present set-up the target is tilted and it makes an angle of 30° with respect to the direction of the incident beam. A 0·5 Ci ^{203}Hg source which emits gamma rays of energy 279 keV is used. Two NaI (Tl) scintillation spectrometers, one for the detection of the scattered gamma rays ($1\frac{1}{2} \times 1\frac{1}{2}$ in^2 crystal) and the other for the fluorescent K x rays ($2 \times \frac{1}{2}$ in^2 crystal) and a fast–slow coincidence system with an effective resolving time of about 30 ns were employed. The final coincidences were recorded on a twenty channel pulse-height analyser. Both the chance and false count rates were experimentally determined. The false coincidence rate is determined by replacing the target by an aluminium foil of equivalent thickness, that is, with the same number of electrons per square centimetre and subtracting the corresponding chance coincidence rate. Aluminium is used since the electrons in it are loosely bound. The experimental values of the differential cross section ratios were determined at scattering angles of 30°, 50°, 70°, 105° and 125° with a tungsten scatterer of thickness 12·74 mg cm^{-2} and at 30° with an erbium scatterer of 25·5 mg cm^{-2} thickness. The tungsten foil was supplied by the Atomic Fuels Division of the Bhabha Atomic Research Centre, Bombay and the erbium foil by M/s Leico Industries Ltd, New York. Both these foils are of high purity (more than 99·9 %).

3. Results and discussion

The experimental values of the differential cross section ratio ($\mathrm{d}\sigma_\mathrm{K}/\mathrm{d}\sigma_\mathrm{F}$) of bound to free electrons are given in figure 1. The $\mathrm{d}\sigma_\mathrm{K}/\mathrm{d}\sigma_\mathrm{F}$ value increases from 0·267 at an angle of 30° to 0·94 at 125°. The small value for the ratio at lower angles can be easily understood from the fact that little momentum is transferred to the electron at those angles, which results in the low probability for the ejection of the bound electrons. But even at a large scattering angle of 125° the $\mathrm{d}\sigma_\mathrm{K}/\mathrm{d}\sigma_\mathrm{F}$ ratio does not reach unity due to the fact that the incident energy is itself low. In fact with higher incident energies it is observed that the ratio exceeds unity at higher angles of scattering (Krishna Reddy *et al* 1970). No relativistic calculations are available in the literature which can be compared directly with the experimental values throughout the angular range of observation.

Many workers tried to predict the observed behaviour approximately, using Thomas–Fermi, Hartree–Fock and Wentzel models for the charge distribution in the scattering atoms. In all these calculations an incoherent scattering function S_K is introduced to correct the free electron cross section. Then

$$\frac{\mathrm{d}\sigma_\mathrm{K}}{\mathrm{d}\Omega}(k_0, \theta, Z) = S_\mathrm{K}(k_0, \theta, Z)\frac{\mathrm{d}\sigma_\mathrm{F}}{\mathrm{d}\Omega} \tag{1}$$

where $\mathrm{d}\sigma_\mathrm{K}/\mathrm{d}\Omega$ is the bound electron cross section, $\mathrm{d}\sigma_\mathrm{F}/\mathrm{d}\Omega$ the free electron cross section and S_K is the probability that the K-shell electron receives a momentum q from the incident photon. Here k_0 is the incident photon energy, θ the angle of scattering and Z the atomic number of the scattering element. Neglecting transitions to excited discrete states, S_K can be expressed as

$$S_\mathrm{K}(k_0, \theta, Z) = \int_\mathrm{cont} \left| \int \psi_\mathrm{f}^* \, \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \psi_\mathrm{i} \, \mathrm{d}\boldsymbol{r} \right|^2 \mathrm{d}E. \tag{2}$$

Shimizu *et al* (1965) using a nonrelativistic wavefunction of K-shell electron in a hydrogen-like atom for ψ_i and a plane wave for ψ_f obtained the equation for S_K as

$$S_\mathrm{K} = \frac{32\sqrt{2} \times 137^3 (E_\mathrm{max}^{3/2} - E_\mathrm{min}^{3/2})}{3\pi^2 Z^3 (m_0 c^2)^{3/2} (1 + p_0^2/b^2)^4}. \tag{3}$$

In expression (3) m_0 is the rest mass of the electron, c the velocity of light, p_0 the initial momentum of a K-shell electron and b is given by $Z\hbar/a_0$ where a_0 is the first Bohr radius. In its initial bound state the electron has a randomly directed momentum $(2m_0B)^{1/2}$, where B is the binding energy of the electron involved. For a free electron of this momentum, the momentum after Compton scattering will be in the range

$$q = (2m_0B)^{1/2} \pm (2m_0T)^{1/2}$$

where $T = k_0A(1-\cos\theta)/\{1+A(1-\cos\theta)\}$ and $A = k_0/m_0c^2$. The kinetic energy corresponding to this momentum will be in the range

$$E = q^2/2m_0 = T+B\pm 2(TB)^{1/2}.$$

This provides an estimate of E_{max} and E_{min} in expression (3) as

$$E_{max} = T+B+2(TB)^{1/2}$$

and

$$E_{min} = T+B-2(TB)^{1/2}.$$

The values of S_K calculated on the basis of equation (3) are plotted as the lowest curve in figure 1.

In their treatment on Compton scattering, Jauch and Rohrlich (1955) developed the idea that the initial electron is free but not stationary at large scattering angles when the momentum transferred is more than the momentum of the initial electron. Based on this idea they obtained an expression for the scattering of gamma rays by free electrons with nonzero velocity which is given as

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\frac{(1-\beta^2)}{(1-\beta\cos\alpha)^2}\frac{k^2}{k_0^2}\overline{X} \tag{4}$$

where r_0 is the classical radius of the electron, β the ratio of the initial electron velocity to the velocity of light, α the angle between the initial electron and the incident photon, k_0 and k the incident and scattered photon energies respectively and

$$\frac{k}{k_0} = \frac{1-\beta\cos\alpha}{1+(k_0/E_0)(1-\cos\theta)-\beta\cos\alpha'}. \tag{5}$$

Here θ is the angle of scattering, α' the angle between the initial electron and the outgoing photon, E_0 the total energy of the initial electron given by $E_0 = m_0c^2/(1-\beta^2)^{1/2}$ and

$$\cos\alpha' = \cos\alpha\cos\theta+\sin\alpha\sin\theta\cos\phi.$$

The angle ϕ is the angle between the planes formed by K, $\boldsymbol{p}$ and K, K' where K and K' are the photon momentum three-vectors and $\boldsymbol{p}$ is that of the initial electron and

$$\overline{X} = \frac{K}{K'}+\frac{K'}{K}+2\left(\frac{1}{K}-\frac{1}{K'}\right)+\left(\frac{1}{K}-\frac{1}{K'}\right)^2 \tag{6}$$

with

$$K = E_0k_0(1-\beta\cos\alpha)$$

and

$$K' = E_0k_0(1-\beta\cos\alpha)+kk_0(\cos\theta-1).$$

Here for a particular scattering angle θ, all the values of α and ϕ in the region from 0° to 180° are equally probable and so the average cross section $\langle d\sigma_K/d\Omega\rangle_{av}$ which is a good approximation is calculated. This expression is given as

$$\left\langle\frac{d\sigma_K}{d\Omega}\right\rangle_{av} = \frac{\int_1^{-1}\int_1^{-1} d\sigma_K/d\Omega \, d(\cos\phi)\, d(\cos\alpha)}{\int_1^{-1}\int_1^{-1} d(\cos\phi)\, d(\cos\alpha)}. \tag{7}$$

The average differential cross section using equation (7) is obtained by computing $d\sigma_K/d\Omega$ for different values of ϕ and α on an IBM computer type 1620 and the mean value is determined for each scattering angle. The values thus obtained are shown as the chain curve in figure 1 and are not in agreement with the experimental results even at large scattering angles.

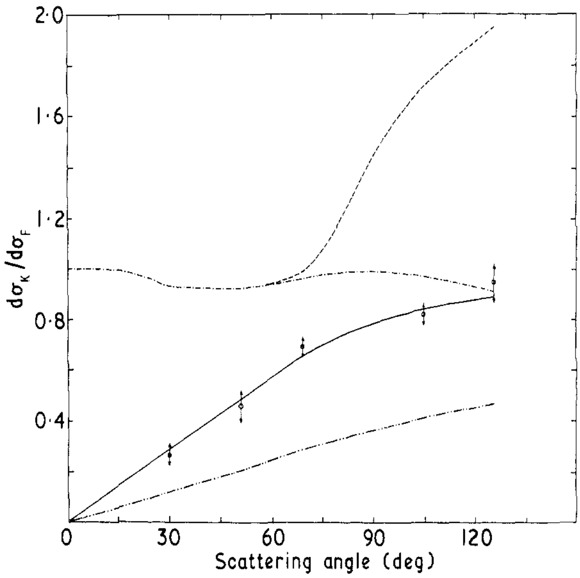

Figure 1. Differential scattering cross section ratio $d\sigma_K/d\sigma_F$ as a function of scattering angle. ——— Experimental curve; – – – – – theoretical curve based on α obtained using equation (8); –·–·–·–·– theoretical curve based on α' obtained using equation (7); –··–··–··– theoretical curve based on S_K obtained using equation (3).

On the other hand, if we ignore the variation in ϕ and consider the variation in α only, then the term $\beta\cos\alpha'$ in the denominator of equation (5) becomes $\beta\cos\alpha$ and the other terms remain the same. Under these conditions as Motz and Missoni (1961) have done, the average cross section can be expressed as

$$\left\langle\frac{d\sigma_K}{d\Omega}\right\rangle_{av} = \frac{\int_1^{-1} d\sigma_K/d\Omega \, d(\cos\alpha)}{\int_1^{-1} d(\cos\alpha)}. \tag{8}$$

The average cross section, obtained by computing $d\sigma_K/d\Omega$ for different values of α between 0° and 180° on the basis of equation (8) and taking the mean value, is plotted for different scattering angles as the broken curve in figure 1. Here also the agreement between theoretical and experimental values is not satisfactory. But, however, in this case both the theoretical and experimental values increase with the angle of scattering beyond 50°. Both the curves due to equations (7) and (8) coincide up to about 50° above which they diverge.

In the low angle region the experimental values are higher than the theoretical values calculated using expression (3). However, there is a tendency for both these values to increase with angle. This feature is observable in the case of Pingot's (1969) results also. This disagreement between theoretical and experimental results may be expected since the nonrelativistic approximation employed in equation (3) is not applicable to K-shell electrons of high Z elements (East and Lewis 1969).

From the above information it is felt that the situation points to the need for a rigorous theory applicable to all angles of scattering.

In table 1 the present experimental results are given along with those obtained by Ramalinga Reddy *et al* (1967) for a tantalum (Z = 73) target using 320 keV gamma rays and those of Pingot (1969) for a gold (Z = 79) scatterer using 279 keV gamma rays.

Table 1. Experimental values of $d\sigma_K/d\sigma_F$

Angle of scattering θ (deg)	Present work $h\nu_0$ = 279 keV tungsten (Z = 74)	erbium (Z = 68)	Pingot (1969) $h\nu_0$ = 279 keV gold (Z = 79)		Ramalinga Reddy *et al* (1967) $h\nu_0$ = 320 keV tantalum (Z = 73)
20			0·206 ± 0·021	(0·251)†	
30	0·267 ± 0·036	0·232 ± 0·013			
45					0·525 ± 0·079
50	0·456 ± 0·062				
55			0·317 ± 0·029	(0·386)†	
60					0·660 ± 0·099
70	0·689 ± 0·040				
90			0·378 ± 0·028	(0·460)†	0·745 ± 0·112
105	0·819 ± 0·042				
110					0·850 ± 0·128
125	0·940 ± 0·080		0·426 ± 0·030	(0·518)†	
160			0·449 ± 0·024	(0·546)†	

† These values are obtained after applying the Z^{-3} variation correction to the gold values of Pingot for easy comparison with the tungsten values of the present authors.

Taking into account the difference in energies, the agreement between the present results and those of Ramalinga Reddy *et al* is quite satisfactory. However, a comparison of the present results with those of Pingot (1969) after correcting for the Z difference (for example Z^{-3} appears in equation (3)), reveals that the agreement in the low angle region is satisfactory while on the large angle side Pingot's results are definitely lower. It may be pointed out in this connection that Pingot's (1968) results using 662 keV gamma rays for the gold target are also significantly less than those of other workers (Motz and Missoni 1961, Varma and Eswaran 1962, Shimizu *et al* 1965, Krishna Reddy 1971). In fact the experimental results of the earlier workers using 662 keV gamma rays are significantly greater than those calculated on the basis of S_K (using equation (3)). The authors find that Pingot's results are consistently smaller with both 662 keV and 279 keV gamma rays than those of the other workers.

Measurements are made for the spectral distribution of the 279 keV gamma rays scattered by K-shell electrons of tungsten at a scattering angle of 125°. The procedure adopted to measure the spectral distribution is basically the same as that employed in the case of the angular distribution and consists of the measurement of the coincidence between gamma rays scattered by K-shell electrons and the fluorescent K x rays. In order to obtain the true gamma ray distribution from the pulse height distributions the coincidence spectra were corrected for the chance and false events as described earlier and the corrected coincidence spectrum is given by the full curve (curve B) in figure 2. The singles gamma ray spectrum (ie, distribution due to free electrons) obtained with an aluminium target is shown in figure 2 by the dotted curve (curve A), for comparison.

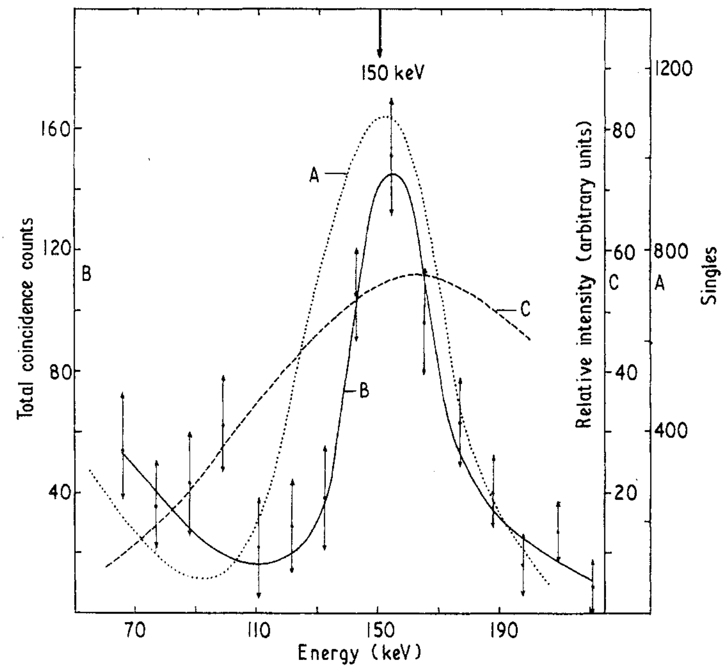

Figure 2. Spectral distribution of 279 keV gamma rays scattered by K-shell electrons of tungsten at $\theta = 125°$. Curve A: distribution due to free electrons (experimental); curve B: distribution due to K-shell electrons (experimental) after correcting for false and chance events; curve C: distribution due to K-shell electrons (theoretical based on equation (9) due to Schnaidt).

No correction has been applied for the finite energy resolution of the scintillation counters for both these curves. The peak of the energy spectrum of the gamma rays scattered by K-shell electrons is close to the Compton energy as predicted by Lambert *et al* (1966). The presence of a high energy tail is also noticeable in the energy distribution due to bound electrons. This feature is in agreement with the result of East and Lewis (1969).

Using hydrogen-like wavefunctions, which is a reasonable approximation for a K-electron, the probability of incoherent scattering (Schnaidt 1934, Sujkowski and Nagel (1961) of a photon into the energy interval dk and the solid angle dΩ about a direction θ is given by

$$\sigma_K(\theta, k)\,d\Omega\,dk = \frac{r_0^2}{2}(1+\cos^2\theta)\frac{256}{3}a^6\frac{k}{k_0}\frac{q^2(a^2+p'^2+3q^2)}{\{a^2+(p'+q)^2\}^3\{a^2+(p'-q)^2\}^3}$$
$$\times\exp\left(-\frac{2a}{p'}\arctan\frac{2p'a}{a^2+q^2-p'^2}\right)\left\{1-\exp\left(-\frac{2\pi a}{p'}\right)\right\}^{-1}d\Omega\,dk. \quad (9)$$

In this expression k_0 and k are the incident and scattered photon energies expressed in terms of m_0c^2, $q = (k_0^2+k^2-2k_0k\cos\theta)^{1/2}$ is the momentum transferred to the atom, p' is the momentum of the recoil electron and is given by $p'^2/2 = k_0-k-a^2/2$, $a = \alpha Z$ where α is the fine structure constant. Both q and p' are expressed in units of m_0c.

Using expression (9) the spectral distribution of the gamma rays scattered by the K-shell electrons of tungsten is calculated and presented by the broken curve (curve C) in figure 2. This curve indicates only the trend of the peak position and does not represent the actual distribution. A detailed study of the spectral distribution will be published elsewhere.

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